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Double Higgs production and quadratic divergence cancellation in little Higgs models with T-parity

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Abstract: We analyze double Higgs boson production at the Large Hadron Collider in the context of Little Higgs models. In double Higgs production, the diagrams involved are directly related to those that cause the cancellation of the quadratic divergence of the Higgs self-energy, providing a robust prediction for this class of models. We find that in extensions of this model with the inclusion of a so-called T-parity, there is a significant enhancement in the cross sections as compared to the Standard Model.

Keywords: Beyond Standard Model, Higgs Physics, Hadron-Hadron Scattering.

Contents

1. Introduction

The presence of quadratic divergences in loop corrections to the scalar Higgs boson self-energy is responsible for the so-called hierarchy problem of the Standard Model (SM); namely, there is no natural way of having a "light" mass (i.e. $\sim 10^2$ GeV) for the Higgs given that loop corrections induce contributions to the mass of the order of the GUT scale –or in general the high energy scale above which new physics enters and the SM ceases to be an effective theory. In supersymmetric extensions of the SM this problem is absent since the bosons are protected from quadratic divergences by their relation to supersymmetric (fermion) partners[1]. The hierarchy problem is also absent in models in which the electroweak symmetry is dynamically broken, since the scalar particles are not fundamental but composite in those cases [2].

Recently a new kind of model was proposed which can solve the hierarchy problem of the scalar Higgs boson. In these models, called Little Higgs (LH) models [3], the Higgs boson is a pseudo-Goldstone boson and its mass is protected by a global symmetry and, unlike supersymmetry, quadratic divergence cancellations are due to contributions from new particles with the same spin.

The phenomenology of these models has been discussed with respect to indirect effects on precision measurements [4] and direct production of the new particles introduced [5].

Since these early contributions, several variations in the LH framework have been proposed [6]. However, the cancellation of quadratic divergences is inherent to any LH model and this requires definite relations among certain couplings. Therefore, any process that involves exclusively these couplings is a robust prediction of the LH mechanism regardless of model variations.

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In this article we study a process that has ingrained in it the cancellation of quadratic divergences of top-quark loops, namely double Higgs production. In section 2 we review the model and the derivation of the masses and couplings of the relevant particles by appropriate diagonalization of mass matrices and show the cancellation of quadratic divergences directly in the broken symmetry phase. In section 3 we derive the amplitudes for double Higgs production at the Large Hadron Collider (LHC) and compute the cross sections. Our main results are presented in section 4. In section 5 we estimate the possible contributions from new T-odd fermions and we conclude in section 6.

2. Masses, couplings and quadratic divergences in the littlest Higgs model

There are many variations of Little Higgs models today, which differ in the symmetry groups and representations of the scalar multiplets, but they all have in common a mechanism of cancellation of the quadratic divergence for the mass of the lightest remaining scalars at one loop order. After the spontaneous breakdown of a global underlying symmetry at a scale $4\pi f$ (supposedly not much higher than a few TeV to avoid fine tuning), the model contains a large multiplet of pseudo-Goldstone bosons, which includes the SM Higgs doublet. While most members of the multiplet receive large masses (again, of a few TeV), the mass of the Higgs boson is protected from quadratic divergences at one loop, and therefore remains naturally smaller. The cancellation is related to the existence of an extra (heavier) top-like quark and its interactions with the scalar sector, feature which is common to all LH models. Consequently, a good test to distinguish a little Higgs from other cases should be based on a signal sensitive to this particular feature of divergence cancellation and rather insensitive to other features. The Higgs pair production at LHC is one of such signals, since it is based on exactly the same diagrams that enter the quadratic divergence cancellation (figure 1), except for the insertion of two gluons (figure 2 and 3).

In order to work out the details, we make use of the Littlest Higgs model, which is a simple case but contains all the necessary ingredients. After spontaneous breakdown of the (high-energy) underlying symmetry, the Little Higgs lagrangian below the scale $4\pi f$ [7] can be written as a non-linear sigma model based on a coset $\mathrm{SU}(5)/\,\mathrm{SO}(5)$ symmetry:

$$\mathcal{L}_\Sigma = \frac{1}{2}\frac{f^2}{4}\mathrm{Tr}|\mathcal{D}_\mu\Sigma|^2, \tag{2.1}$$

where the subgroup $[\mathrm{SU}(2)\times\mathrm{U}(1)]^2$ of $\mathrm{SU}(5)$ is promoted to a local gauge symmetry. The covariant derivative is defined as

$$\mathcal{D}_\mu\Sigma = \partial_\mu\Sigma - i\sum_{j=1}^{2}\left(g_j(W_j\Sigma+\Sigma W_j^T)+g'_j(B_j\Sigma+\Sigma B_j^T)\right). \tag{2.2}$$

To exhibit the interactions, one can expand Σ in powers of $1/f$ around its vacuum expectation value Σ_0

$$\Sigma = \Sigma_0 + \frac{2i}{f}\begin{pmatrix} \phi^\dagger & \frac{h^\dagger}{\sqrt{2}} & \mathbf{0}_{2\times 2} \\ \frac{h^*}{\sqrt{2}} & 0 & \frac{h}{\sqrt{2}} \\ \mathbf{0}_{2\times 2} & \frac{h^T}{\sqrt{2}} & \phi \end{pmatrix} + \mathcal{O}\left(\frac{1}{f^2}\right), \tag{2.3}$$

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where h is the doublet that will remain light and ϕ is a triplet under the unbroken SU(2). The non-zero vacuum expectation value of the field $\langle\Sigma\rangle = \Sigma_0$ leads to the breaking of the global SU(5) symmetry to SO(5) and also breaks the local gauge symmetry $[\mathrm{SU}(2)\times\mathrm{U}(1)]^2$ into its diagonal subgroup, which is identified with the standard model $SU_L(2)\times U_Y(1)$ symmetry group. Following the notation of Han *et al.* [5], we will denote the usual standard model gauge bosons mass eigenstates as $W_L^{\pm}$, Z_L and A_L, where the subscript L denotes light in order to distinguish from the heavy states with mass of order f, denoted by $W_H^{\pm}$, Z_H and A_H.

The standard model fermions acquire their masses via the usual Yukawa interactions. However, in order to cancel the top quark quadratic contribution to the Higgs self-energy, a new-vector like color triplet fermion pair, $\tilde{t}$ and $\tilde{t}'^c$, with quantum numbers $(\mathbf{3},\mathbf{1})_{Y_i}$ and $(\bar{\mathbf{3}},\mathbf{1})_{-Y_i}$ must be introduced. Since they are vector-like, they are allowed to have a bare mass term which is *chosen* such as to cancel the quadratic divergence above scale f.

The coupling of the standard model top quark to the pseudo-Goldstone bosons and the heavy colored fermions in the littlest Higgs model is chosen to be

$$\mathcal{L}_Y = \frac{1}{2}\lambda_1 f \epsilon_{ijk}\epsilon_{xy}\chi_i\Sigma_{jx}\Sigma_{ky}u_3'^c + \lambda_2 f\tilde{t}\tilde{t}'^c + \mathrm{h.c.}, \tag{2.4}$$

where $\chi_i = (b_3, t_3, \tilde{t})$ and ϵ_{ijk} and ϵ_{xy} are antisymmetric tensors. The new model parameters λ_1, λ_2 are supposed to be of the order of unity.

The linearized part of eq. (2.4) describing the third generation mass terms and interactions with the neutral Higgs field, denoted by h^0, (before spontaneous symmetry breaking, SSB), is given by:

$$\mathcal{L}_t = \lambda_2 f\tilde{t}\tilde{t}'^c - i\lambda_1\sqrt{2}t_3 h^0 u_3'^c + \lambda_1 f\tilde{t}u_3'^c - \frac{\lambda_1}{f}\tilde{t}h^0 h^{0*}u_3'^c + h.c. \tag{2.5}$$

After SSB, we write $h^0 = 1/\sqrt{2}(v+H)$, and follow Perelstein *et al.* [8] in defining left handed fields $t_{3L}\equiv t_3$, $\tilde{t}_L \equiv \tilde{t}$ and right handed fields $\bar{u}'_{3R}\equiv u_3'^c$, $\bar{\tilde{t}}_R \equiv \tilde{t}'^c$ to obtain

$$\begin{aligned}\mathcal{L}_t = &\begin{pmatrix}\bar{u}'_{3R} & \bar{\tilde{t}}_R\end{pmatrix}\begin{pmatrix}-i\lambda_1 v & \lambda_1 f(1-v^2/2f^2)\\ 0 & \lambda_2 f\end{pmatrix}\begin{pmatrix}t_{3L}\\ \tilde{t}_L\end{pmatrix} - i\lambda_1 H\bar{u}'_{3R}t_{3L}\\ &-\lambda_1\frac{v}{f}H\bar{u}'_{3R}\tilde{t}_L - \frac{\lambda_1}{2f}H^2\bar{u}'_{3R}\tilde{t}_L + h.c.\end{aligned} \tag{2.6}$$

In order to leave the fermion mass term in its standard form we make the field redefinitions $t_{3L}\to -it_{3L}$ and $\tilde{t}_L \to -\tilde{t}_L$ in the left-handed fields resulting in the following lagrangian:

$$\begin{aligned}\mathcal{L}_t = -&\begin{pmatrix}\bar{u}'_{3R} & \bar{\tilde{t}}_R\end{pmatrix}\begin{pmatrix}\lambda_1 v & \lambda_1 f(1-v^2/2f^2)\\ 0 & \lambda_2 f\end{pmatrix}\begin{pmatrix}t_{3L}\\ \tilde{t}_L\end{pmatrix} - \lambda_1 H\bar{u}'_{3R}t_{3L}\\ &+\lambda_1\frac{v}{f}H\bar{u}'_{3R}\tilde{t}_L + \frac{\lambda_1}{2f}H^2\bar{u}'_{3R}\tilde{t}_L + h.c.\end{aligned} \tag{2.7}$$

Diagonalizing the mass matrix

$$M = \begin{pmatrix}\lambda_1 v & \lambda_1 f(1-v^2/2f^2)\\ 0 & \lambda_2 f\end{pmatrix} \tag{2.8}$$

we obtain the usual result for the eigenvalues corresponding to the top quark t and the heavy top T which are, up to order $\mathcal{O}(v/f)$:

$$m_t = \frac{\lambda_1\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}}\, v\ ; \quad m_T = \sqrt{\lambda_1^2+\lambda_2^2}\, f. \tag{2.9}$$

In our numerical code, we will use as input the values of m_T and f, from which one obtains the required values of λ_1 and λ_2:

$$\lambda_{1(2)}^2 = \frac{m_T^2}{2f^2}\left(1+(-)\sqrt{1-4\frac{m_t^2 f^2}{v^2 m_T^2}}\right). \tag{2.10}$$

From equation (2.10), one clearly sees that there is a condition relating top masses and v.e.v.'s that these models impose:

$$m_T > 2\frac{m_t v}{f} \simeq \sqrt{2} f \tag{2.11}$$

and which we incorporate in our analysis. The relevant couplings between Higgs and top quarks are obtained in a straightforward manner after diagonalization of eq. (2.7) and are given by (the corresponding vertices are obtained via multiplication by $(-i)$):

$$\begin{aligned}
g_{Htt} &= \frac{\lambda_1\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}} \qquad (2.12)\\
g_{HT_Rt_L} &= \frac{\lambda_1^2}{\sqrt{\lambda_1^2+\lambda_2^2}}\\
g_{HTT} &= -\frac{\lambda_1^2\lambda_2^2}{(\lambda_1^2+\lambda_2^2)^{3/2}}\,\frac{v}{f}\\
g_{HHTT} &= -\frac{1}{f}\frac{\lambda_1^2}{\sqrt{\lambda_1^2+\lambda_2^2}}\\
g_{HHtt} &= \frac{m_t}{f^2}\frac{\lambda_1^2}{\lambda_1^2+\lambda_2^2-4fv'/v^2}\\
g_{Ht_RT_L} &= -\frac{\lambda_1\lambda_2^3}{(\lambda_1^2+\lambda_2^2)^{3/2}}\,\frac{v}{f}
\end{aligned}$$

The relevant Feynman diagrams for the Higgs self-energy are shown in figure 1.

The cancellation of tadpole diagrams requires that

$$g_{Htt} m_t + g_{HTT} m_T = 0 \tag{2.13}$$

whereas the cancellation of higgs self-energy quadratic divergences implies

$$g_{Htt}^2 + g_{HTT}^2 + g_{HT_Rt_L}^2 + g_{Ht_RT_L}^2 + g_{HHtt}\, m_t + g_{HHTT}\, m_T = 0 \tag{2.14}$$

These conditions are satisfied up to terms of order $\mathcal{O}(v/f)$ by the masses and couplings listed above.

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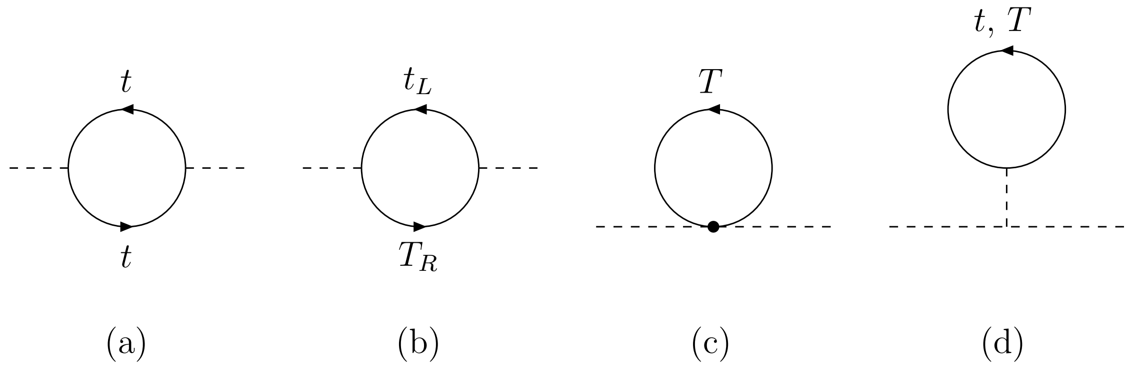

Figure 1: One-loop corrections to the Higgs mass, to order v/f: (a) standard top quark loop, (b) mixture of standard and extra top quark loop, (c) extra top quark loop with a 4-particle vertex, and (d) tadpoles with standard and with extra top quark loops. There are other diagrams but they are suppressed by factors of order $(v/f)^2$ or higher.

In the simplest LH models, strict bounds on the parameters are obtained. In particular, electroweak precision constraints require $f > 3.5$ TeV [4]. However, in a recent variation on the littlest Higgs model, where a so-called T-parity that interchanges the two subgroups $[\mathrm{SU}(2) \times \mathrm{U}(1)]_1$ and $[\mathrm{SU}(2) \times \mathrm{U}(1)]_2$ of SU(5) is introduced, can significantly lower this bound to $f > 500$ GeV [9, 10]. This is an important point for the phenomenology of these models, since a lower f implies larger deviations from the SM. Since the T-odd states do not participate in the cancellation of quadratic divergences, our calculation is valid in models with T-parity as well. T-parity also forbids the generation of a vacuum expectation value for the triplet scalar field (i.e., $v' = 0$ in the notation of T. Han et al. [5]), which is one of the causes for easing the electroweak constraints.

3. Amplitudes for double Higgs production

We now turn to Higgs boson production at the LHC in the LH model, which involves the very same couplings responsible for the cancellation of quadratic divergences.

Gluon-gluon fusion is the dominant mechanism for SM Higgs boson pair production at the LHC [11]. The amplitude for $gg \to HH$ is dominated by top quark loops, in the form of triangle and box diagrams. figures 2 and 3 show the case for a LH model. The SM case is similar, except that figure 2.a and all extra heavy-top loops are absent. We also would like to point out that T-parity forbids a term like $hh\phi$ in the radiatively generated Coleman-Weinberg potential. Therefore, there is no contribution of the heavy scalar in figure 2.b and the trilinear Higgs coupling is the same as in the SM.

Let us now write the expressions for the amplitude. In what follows, the external momenta p_a, p_b, p_c, p_d are defined as incoming. The contribution from triangle diagrams

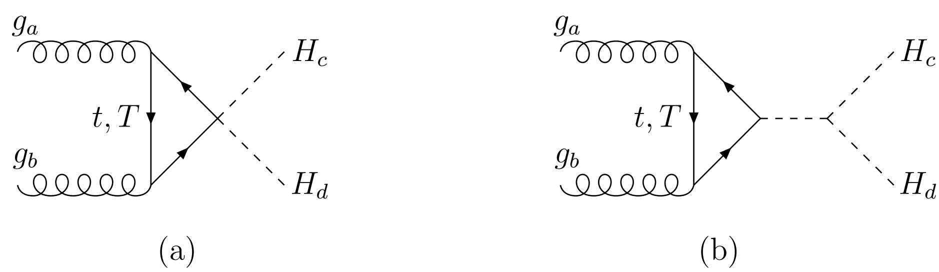

Figure 2: Triangle contributions to Higgs boson pair production at LHC in a Little Higgs model.

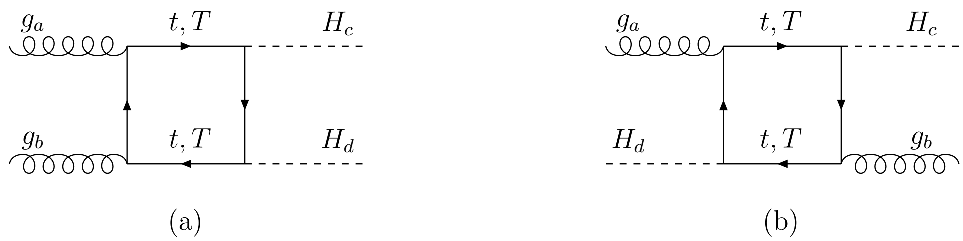

Figure 3: Box contributions to Higgs boson pair production at LHC in a Little Higgs model.

is given by:

$$i\mathcal{M}^{\triangle}(g_a g_b \to H_c H_d) = -\frac{\alpha_s}{4\pi^3}\delta_{AB}\left(g_{Htt}I(m_t)\frac{g_{HHH}}{\hat{s}-M_H^2-iM_H\Gamma_H}\right.$$
$$\left.+g_{HTT}I(m_T)\frac{g_{HHH}}{\hat{s}-M_H^2-iM_H\Gamma_H}+g_{HHtt}I(m_t)+g_{HHTT}I(m_T)\right) \tag{3.1}$$

where the integral $I(m_Q)$ is:

$$I(m_Q) = \int d^4q\, \frac{Tr\,[(\not{q}+m_Q)\gamma^\mu(\not{q}+\not{p}_a+m_Q)\gamma^\nu(\not{q}+\not{p}_a+\not{p}_b+m_Q)]}{[q^2-m_Q^2][(q+p_a)^2-m_Q^2][(q+p_a+p_b)^2-m_Q^2]}\epsilon_\mu(p_a)\epsilon_\nu(p_b). \tag{3.2}$$

This integral reduces to the following result:

$$I(m_Q) = i\,4\pi^2\,m_Q[1+(2m_Q^2-\hat{s}/2)C_0(0,0,\hat{s},m_Q^2,m_Q^2,m_Q^2)]\epsilon(p_a)\cdot\epsilon(p_b), \tag{3.3}$$

where C_0 is the scalar Passarino-Veltman integral[12] defined as:

$$C_0 = \int \frac{d^4q}{i\pi^2}\,\frac{1}{[q^2-m_Q^2][(q+p_a)^2-m_Q^2][(q+p_a+p_b)^2-m_Q^2]} \tag{3.4}$$

The contribution from box diagrams can be written as:

$$i\mathcal{M}^{\Box}(g_a g_b \to H_c H_d) = -\frac{\alpha_s}{8\pi^3}\delta_{AB}\left(g_{Htt}^2 I_1(m_t)+g_{HTT}^2 I_1(m_T)+I_2(m_t,m_T)\right) \tag{3.5}$$

Because the Higgs vertex is not diagonal in the (t,T) flavor, there are two types of boxes. The function I_1 comes from boxes with either only standard top quarks or only extra heavy-top quarks in them, whereas the function I_2 comes from boxes with both tops and extra heavy-tops.

There are two basic box diagrams, planar (figure 3.a) and non-planar (figure 3.b), according to whether the Higgses are adjacent in the loop or not. A planar box with a single type of quark is given by:

$$
\begin{aligned}
&I_1^P(m_Q) = \\
&= \int d^4q \, \frac{Tr\left[(\not{q} + m_Q)\gamma^\mu(\not{q} + \not{p}_a + m_Q)\gamma^\nu(\not{q} + \not{p}_a + \not{p}_b + m_Q)(\not{q} + \not{p}_a + \not{p}_b + \not{p}_c + m_Q)\right]}{[q^2 - m_Q^2][(q+p_a)^2 - m_Q^2][(q+p_a+p_b)^2 - m_Q^2][(q+p_a+p_b+p_c)^2 - m_Q^2]} \\
&\quad \times \epsilon_\mu(p_a)\epsilon_\nu(p_b) + \Big\{p_a \leftrightarrow p_b\Big\} + \Big\{p_c \leftrightarrow p_d\Big\} + \Big\{p_a \leftrightarrow p_b,\, p_c \leftrightarrow p_d\Big\}, \qquad (3.6)
\end{aligned}
$$

while a non-planar box is:

$$
\begin{aligned}
&I_1^{NP}(m_Q) = \\
&= \int d^4q \, \frac{Tr\left[(\not{q} + m_Q)\gamma^\mu(\not{q} + \not{p}_a + m_Q)(\not{q} + \not{p}_a + \not{p}_c + m_Q)\gamma^\nu(\not{q} + \not{p}_a + \not{p}_b + \not{p}_c + m_Q)\right]}{[q^2 - m_Q^2][(q+p_a)^2 - m_Q^2][(q+p_a+p_c)^2 - m_Q^2][(q+p_a+p_b+p_c)^2 - m_Q^2]} \\
&\quad \times \epsilon_\mu(p_a)\epsilon_\nu(p_b) \quad + \Big\{p_a \leftrightarrow p_b\Big\} \qquad (3.7)
\end{aligned}
$$

The total contribution for boxes with a single type of quark is then

$$
I_1(m) = I^P(m) + I^{NP}(m) \qquad (3.8)
$$

We also have to compute the contribution of box diagrams with both t and T running in the loop. There are also planar and a non-planar contributions in this case. For the planar contribution we have:

$$
\begin{aligned}
&I_2^P(m_t, m_T) = \\
&= \int d^4q \frac{Tr\left[(\not{q} + m_t)\gamma^\mu(\not{q}+\not{p}_a + m_t)\gamma^\nu(\not{q}+\not{p}_a+\not{p}_b + m_t)\left(g_{HT_Rt_L}\frac{1+\gamma_5}{2} + g_{Ht_RT_L}\frac{1-\gamma_5}{2}\right)\right]}{[q^2 - m_t^2][(q+p_a)^2 - m_t^2][(q+p_a+p_b)^2 - m_t^2]} \\
&\times \frac{\left[(\not{q} + \not{p}_a + \not{p}_b + \not{p}_c + m_T)\left(g_{HT_Rt_L}\frac{1-\gamma_5}{2} + g_{Ht_RT_L}\frac{1+\gamma_5}{2}\right)\right]}{[(q+p_a+p_b+p_c)^2 - m_T^2]} \\
&\times \epsilon^\mu(p_a)\epsilon^\nu(p_b) + \Big\{p_a \leftrightarrow p_b\Big\} + \Big\{p_c \leftrightarrow p_d\Big\} + \Big\{p_a \leftrightarrow p_b,\, p_c \leftrightarrow p_d\Big\} \qquad (3.9)
\end{aligned}
$$

and the non-planar contribution is written as:

$$
\begin{aligned}
&I_2^{NP}(m_t, m_T) = \\
&= \int d^4q \frac{Tr\left[(\not{q} + m_t)\gamma^\mu(\not{q}+\not{p}_a+m_t)\left(g_{HT_Rt_L}\frac{1+\gamma_5}{2}+g_{Ht_RT_L}\frac{1-\gamma_5}{2}\right)(\not{q} + \not{p}_a+\not{p}_b + m_t)\gamma^\nu\right]}{[q^2 - m_t^2][(q+p_a)^2 - m_t^2][(q+p_a+p_b)^2 - m_t^2]} \\
&\times \frac{\left[(\not{q} + \not{p}_a + \not{p}_b + \not{p}_c + m_T)\left(g_{HT_Rt_L}\frac{1-\gamma_5}{2} + g_{Ht_RT_L}\frac{1+\gamma_5}{2}\right)\right]}{[(q+p_a+p_b+p_c)^2 - m_T^2]} \\
&\times \epsilon^\mu(p_a)\epsilon^\nu(p_b) + \Big\{p_a \leftrightarrow p_b\Big\} \qquad (3.10)
\end{aligned}
$$

Accordingly, the total contribution for boxes with both t and T is given by:

$$
I_2 = I_2^P(m_t, m_T) + I_2^P(m_T, m_t) + I_2^{NP}(m_t, m_T) + I_2^{NP}(m_T, m_t). \qquad (3.11)
$$

We can then express these integrals in terms of Passarino-Veltman functions, in a way analogous to eqs. (3.2) and (3.3). We computed these transformations using the package FeynCalc [13]. Since this procedure is straightforward and the final result is rather long, we did not include the expressions here.

4. Cross section results

Using the total scattering amplitude $\mathcal{M}(g_a g_b \to H_c H_d) = \mathcal{M}^{\triangle} + \mathcal{M}^{\square}$ we can build the partonic differential cross section:

$$\frac{d\hat{\sigma}}{d\Omega} = \frac{1}{128\pi^2 \hat{s}} \sqrt{1 - 4M_H^2/\hat{s}}\, \overline{|\mathcal{M}|^2}. \tag{4.1}$$

We must point out that we have included a factor of 1/2 due to the identical particles in the final state. Consequently, to obtain the total cross section one must integrate eq. (4.1) over the whole 4π solid angle. Here $\overline{|\mathcal{M}|^2}$ is the squared matrix element averaged over initial color and helicity states:

$$\overline{|\mathcal{M}|^2} = \frac{1}{32} \sum_{i,j=1,2} |\mathcal{M}_{ij}|^2 \tag{4.2}$$

where the sum is over the two physical gluon polarizations.

We performed the calculation in the center-of-momentum frame of the gluons. In that case, the transversality condition of the gluon polarization vectors also implies $p_a \cdot \epsilon(p_b) = p_b \cdot \epsilon(p_a) = 0$. Therefore we use the following parametrization (recalling that all 4-momenta were defined as incoming):

$$\begin{aligned} p_a &= (\sqrt{\hat{s}}/2, 0, 0, \sqrt{\hat{s}}/2) \\ p_b &= (\sqrt{\hat{s}}/2, 0, 0, -\sqrt{\hat{s}}/2) \\ p_c &= (-\sqrt{\hat{s}}/2, 0, -p\sin\theta, -p\cos\theta) \\ p_d &= (-\sqrt{\hat{s}}/2, 0, p\sin\theta, p\cos\theta) \\ \epsilon^{(1)} &= (0, 1, 0, 0) \\ \epsilon^{(2)} &= (0, 0, 1, 0) \end{aligned} \tag{4.3}$$

where the Higgs boson center-of-mass momentum is $p = \sqrt{\hat{s}/4 - M_H^2}$. We numerically integrate the Passarino-Veltman functions using the package LoopTools [14]. Finally, we obtain the $pp \to HH$ cross section at the LHC by convoluting the partonic cross section with the gluon distribution function:

$$\begin{aligned} \sigma(pp \to HH) = \frac{K}{2} \int\ & dx_1 dx_2\, [g_1(x_1, Q^2) g_2(x_2, Q^2) + g_2(x_1, Q^2) g_1(x_2, Q^2)] \\ & \hat{\sigma}(gg \to HH)\theta(x_1 x_2 s - 4M_H^2), \end{aligned} \tag{4.4}$$

where we used the Set 3 of CTEQ6 leading gluon distribution function with momentum scale $Q^2 = \hat{s}$ [15]. A $K = 2$ factor was included to take into account QCD corrections [16].

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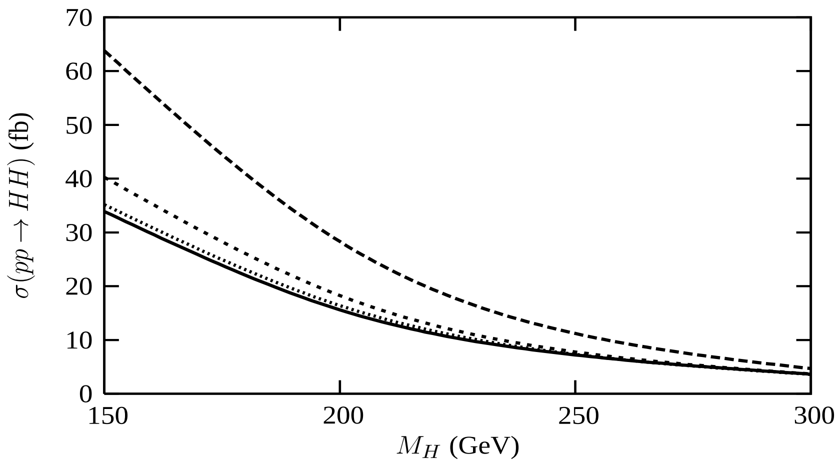

Figure 4: Cross section for double Higgs production at the LHC for $M_T = 4$ TeV and $f = 500$ GeV (dashed line), 1000 GeV (short dashed line) and 2000 GeV (dotted line). In solid line is shown the SM result.

In figure 4 we plot the cross section for the double Higgs production process at the LHC for fixed $M_T = 4$ TeV, a Higgs boson mass in the range 150–300 GeV and for $f = 500$, 1000 and 2000 GeV. As expected, we find that the largest deviations from the SM result occurs for small Higgs boson mass and small decay constant f. In this sense it is important to consider models with T-parity, where f is not required to be too large. Our results are otherwise consistent with the authors of Ref. [17], where values around $f = 3.5$ TeV are used. Since, as we show below, the cross section for double Higgs production is almost independent of the heavy top mass for $m_T > 2f$, we could have chosen a somewhat smaller value of m_T without significantly changing the result.

In order to explore the dependence on the heavy top quark mass in figure 5 we plot the cross section for the double Higgs production process at the LHC for fixed $M_H = 200$ GeV and $f = 1000$ GeV as a function of m_T. We can see that the result grows with m_T, reaching a constant limit for m_T above ~ 2.5 TeV. The growth in the cross section as m_T increases is a consequence of non-decoupling of the heavy T quark [18].

We also include an analysis of the significance of the signals, which defined as:

$$\text{significance} = \frac{\mathcal{L} \cdot \sigma_{LH} - \mathcal{L} \cdot \sigma_{SM}}{\sqrt{\mathcal{L} \cdot \sigma_{SM}}}, \tag{4.5}$$

where $\mathcal{L}$ is the integrated luminosity of LHC and $\sigma_{LH},\ \sigma_{SM}$ are the two-Higgs production cross sections for pp collisions in the Little Higgs and Standard models, respectively. Assuming a luminosity of $\mathcal{L} = 10\ \text{fb}^{-1}$ for the LHC, we obtain the results shown in figure 6. Of course the significance scales as $\sqrt{\mathcal{L}}$ and the figure can be easily used for larger LHC integrated luminosities.

A word of caution is necessary at this point. It is beyond the scope of this work to include an analysis of the simulation of detector sensitivity and possible backgrounds needed for a realistic study of double Higgs production and detection in the LH model,

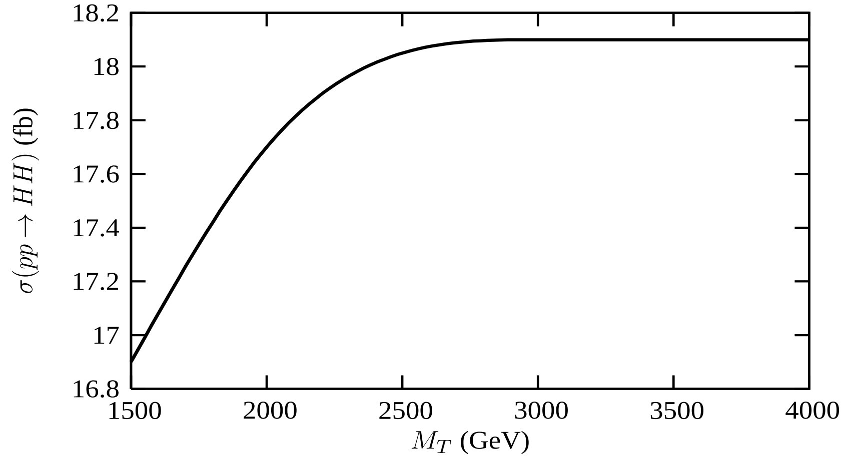

Figure 5: Cross section for double Higgs production at the LHC for $M_H = 200$ GeV and $f = 1000$ GeV, as a function of the heavy top quark mass M_T.

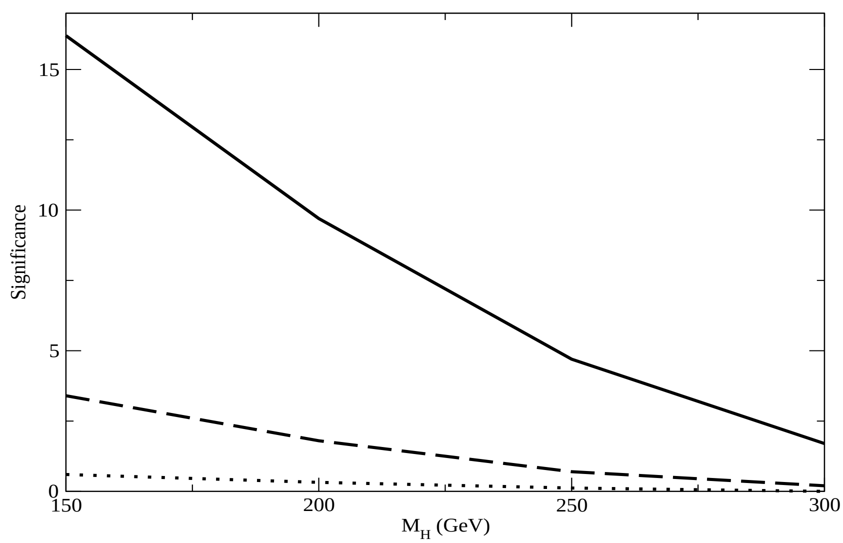

Figure 6: Significance of the two-Higgs signal in the Little Higgs model with respect to the Standard model, at the LHC, assuming an integrated luminosity of 10 fb^{-1}. Solid, dashed and dotted lines correspond to $f = 500$, 1000 and 2000 GeV, respectively.

which was done in the SM in [19]. Our study is just a first step in gauging how different from the SM the LH signal for double Higgs production is.

5. Contributions from T-odd fermions

In LH models with T-parity, the SM fermion doublet spectrum must be doubled in order to avoid dangerous contributions to four fermion operators [9]. These two doublets are exchanged under T-parity and one can construct a T-even and a T-odd linear combination, where the T-even combination is identified with the SM fermion doublet. In order to give a large mass to the T-odd combination while leaving the T-even combination massless, one must introduce a set of new T-odd "mirror" fermions.

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The T-invariant interaction that generates the mass terms is somewhat model-dependent. For instance, in an extension of the simplest SU(5)/ SO(5) LH model to $\mathrm{SU}(5)_L \times \mathrm{SU}(5)_R/\ \mathrm{SO}(5)_V$ Low demonstrated that it is possible to generate heavy masses for all T-odd fermions without introducing new SM Higgs couplings [9]. Therefore, in this particular extension our results would be unaffected by these new fermions.

However, in the simplest SU(5)/ SO(5) LH model as well as its $\mathrm{SU}(5)_L \times \mathrm{SO}(5)_R/\ \mathrm{SO}(5)_V$ extension, these T-odd fermions may have interactions with the SM Higgs boson and we estimate below their contribution to the Higgs pair production process. We stress that in the top quark sector, already analyzed, all three LH models with T-parity have the same spectrum.

The T-parity and global SU(5) invariant Yukawa-like interaction is given by [9]

$$\mathcal{L}_Y = \kappa f \left(\bar{\Psi}_2 \xi \Psi_c + \bar{\Psi}_1 \Sigma_0 \Omega \xi^\dagger \Omega \Psi_c \right) + h.c.\ , \tag{5.1}$$

where κ is a coupling constant of order one and the fermions Ψ_1, Ψ_2 and Ψ_c are in an SU(5) representation:

$$\Psi_1 = \begin{pmatrix} \psi_1 \\ 0 \\ 0 \end{pmatrix}, \quad \Psi_2 = \begin{pmatrix} 0 \\ 0 \\ \psi_2 \end{pmatrix}, \quad \text{and} \quad \Psi_c = \begin{pmatrix} \tilde{\psi}_c \\ \chi_c \\ \psi_c \end{pmatrix}, \tag{5.2}$$

ξ is related to the exponential of the Goldstone boson fields, $\xi = e^{i\Pi/f}$, $\Omega = \mathrm{diag}(1, 1, -1, 1, 1)$ and

$$\Sigma_0 = \begin{pmatrix} & & \mathbf{1}_{2\times 2} \\ & 1 & \\ \mathbf{1}_{2\times 2} & & \end{pmatrix}. \tag{5.3}$$

Under T-parity the doublets are interchanged as $\psi_1 \leftrightarrow -\psi_2$ and therefore one can construct T-even and T-odd fermion doublets as:

$$\psi_- = \frac{1}{\sqrt{2}}(\psi_1 + \psi_2), \quad \psi_+ = \frac{1}{\sqrt{2}}(\psi_1 - \psi_2), \tag{5.4}$$

and the SM fermion doublet is identified as

$$\psi_+ = -i\sigma_2 \begin{pmatrix} u_L \\ d_L \end{pmatrix} = \begin{pmatrix} -d_L \\ u_L \end{pmatrix}. \tag{5.5}$$

We will also denote

$$\psi_- = -i\sigma_2 \begin{pmatrix} u_{L-} \\ d_{L-} \end{pmatrix} = \begin{pmatrix} -d_{L-} \\ u_{L-} \end{pmatrix}, \quad \tilde{\psi}_c = \begin{pmatrix} -\tilde{d}_c \\ \tilde{u}_c \end{pmatrix} \quad \text{and} \quad \psi_c = \begin{pmatrix} -d_c \\ u_c \end{pmatrix} \tag{5.6}$$

The mass terms are easily obtained from eq. (5.1) by setting $\xi = 1$:

$$\mathcal{L}_{mass} = \sqrt{2}\kappa f (\bar{d}_{L-}\tilde{d}_c + \bar{u}_{L-}\tilde{u}_c) + h.c. \tag{5.7}$$

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As advertised the T-even doublets remain massless at this level, getting their masses only from electroweak symmetry breaking. The T-odd fermions d_{L-}, u_{L-}, $\tilde{d}_c$ and $\tilde{u}_c$ get non-diagonal mass terms of order κf and one must include additional terms to generate large masses to the remaining T-odd ψ_c and χ_c fermions.

In order to study new Higgs boson interactions with T-odd fermions arising from eq. (5.1), we will use only the Higgs boson component in the Goldstone field matrix Π, which we will denote by Π_H:

$$\Pi_H = H/2 \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \tag{5.8}$$

The interaction with one Higgs boson is given by:

$$\mathcal{L}_{1H} = \kappa f \left(\bar{\Psi}_2 \frac{i}{f} \Pi_H \Psi_c - \bar{\Psi}_1 \Sigma_0 \Omega \frac{i}{f} (\Pi_H)^T \Omega \Psi_c \right) + h.c. = i \frac{\kappa}{\sqrt{2}} H \bar{u}_{L-} \chi_c + h.c. \tag{5.9}$$

We are also interested in the interactions with two Higgs bosons:

$$\mathcal{L}_{2H} = -\frac{\kappa}{2f} \left(\bar{\Psi}_2 \Pi_H^2 \Psi_c + \bar{\Psi}_1 \Sigma_0 \Omega (\Pi_H^2)^T \Omega \Psi_c \right) + h.c. = -\frac{\kappa}{2f} H^2 \bar{u}_{L-} (u_c + \tilde{u}_c) + h.c. \tag{5.10}$$

At this point we are neglecting terms suppressed by factors of v/f.

We are now ready to estimate the contributions of these new T-odd fermions to the double Higgs production process. A more detailed analysis of the contributions of new T-odd fermions to single Higgs production was performed recently by Chen, Tobe and Yuan [20]. First notice that due to the proportionality of the new couplings to the heavy fermion mass ($M \simeq \kappa f$) we expect that these contributions will not decouple for large masses, as in the case of the top sector. Secondly, since there is no $u_{L-} - \chi_c$ mixing, the triangle diagram in figure 2b vanishes at this order (there is a contribution at order v/f). However, the contribution of this diagram was subdominant anyway because the off-shellness of the intermediate Higgs boson.

There are contributions from diagrams of the type of figure 2a with fermions u_{L-} and $\tilde{u}_c$ running in the triangle. There are also contributions from box diagrams of the type shown in figure 3 with either one u_{L-} and three χ_c's or three u_{L-}'s and one χ_c running in the box.

Given the above arguments, we estimate that the contribution from the two generations of new T-odd fermions is of the same order as of the top quark sector in the large mass limit of these new fermions. Therefore, the significance of the signal can be greatly increased. If we write $\sigma_{LH} = \sigma_{SM} + \delta\sigma_{LH}$ with $\delta\sigma_{LH} = \delta\sigma_{LH}^{(t+T)} + \delta\sigma_{LH}^{(u_{L-})} + \delta\sigma_{LH}^{(c_{L-})}$ we could expect a factor of roughly three increase in the significance plotted in figure 6. This implies in a much better prospect of finding this signal.

However, we must stress that this analysis is model dependent and the conservative robust result is the one presented in the previous section.

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6. Conclusions

The double Higgs production process probes the nature of the electroweak symmetry breaking mechanism. This process is intimately tied to the cancellation of quadratic divergences in Little Higgs models. Here we have studied the reach of the LHC to probe the LH models in this way. We found that only for relatively small values of the energy scale f, of the order of 500 to 1000 GeV, it is possible to distinguish meaningfully the LH from the SM. These low values are attainable without violating the electroweak precision limits only in models where an extra T parity is incorporated [9, 10]. On the other hand, these results depend only mildly on the heavy top quark mass m_T; while the situation is more promising for larger values of m_T, it becomes practically independent of it for m_T above 2.5 TeV. This process can also be greatly enhanced in models where new T-odd fermions interact with the Higgs boson.

For sure the observation of an enhancement in this channel is not a definitive proof of the Little Higgs as the mechanism chosen by Nature for the electroweak scale stabilization and a detailed study of other channels will be required.

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